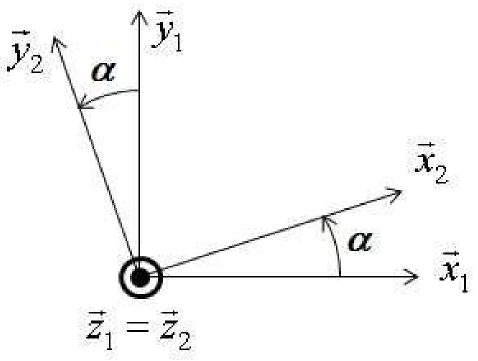
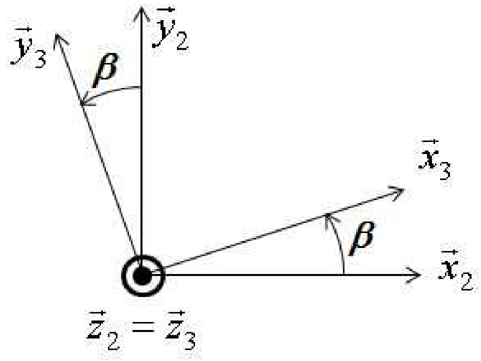
PROJECTION VECTORIELLE
PRODUIT VECTORIEL

Q1)





Q2)

- On a : $\overrightarrow{AB} = a\ \vec{x}_2 = a\times(\cos\alpha\ \vec{x}_1 + \sin\alpha\ \vec{y}_1)$ ⇨ $\overrightarrow{AB} = \begin{pmatrix} a\cos\alpha \\ a\sin\alpha \\ 0 \end{pmatrix}_{B_1}$

- On a : $\overrightarrow{BC} = b\ \vec{x}_3 = b\times(\cos\beta\ \vec{x}_2 + \sin\beta\ \vec{y}_2)$

⇨ $\overrightarrow{BC} = b\times[\cos\beta\times(\cos\alpha\ \vec{x}_1 + \sin\alpha\ \vec{y}_1) + \sin\beta\times(-\sin\alpha\ \vec{x}_1 + \cos\alpha\ \vec{y}_1)]$

⇨ $\overrightarrow{BC} = \begin{pmatrix} b\times(\cos\beta\cos\alpha - \sin\beta\sin\alpha) \\ b\times(\cos\beta\sin\alpha + \sin\beta\cos\alpha) \\ 0 \end{pmatrix}_{B_1}$

Q3) $\overrightarrow{AB}.\ \overrightarrow{BC} = \begin{pmatrix} a\cos\alpha \\ a\sin\alpha \\ 0 \end{pmatrix}_{B_1} . \begin{pmatrix} b\times(\cos\beta\cos\alpha - \sin\beta\sin\alpha) \\ b\times(\cos\beta\sin\alpha + \sin\beta\cos\alpha) \\ 0 \end{pmatrix}_{B_1}$

⇨ $\overrightarrow{AB}.\ \overrightarrow{BC} = a\,b\times(\cos^2\alpha\cos\beta - \cancel{\cos\alpha\sin\beta\sin\alpha} + \sin^2\alpha\cos\beta + \cancel{\sin\alpha\sin\beta\cos\alpha})$

⇨ $\overrightarrow{AB}.\ \overrightarrow{BC} = a\,b\times\cos\beta\times\underbrace{(\cos^2\alpha + \sin^2\alpha)}_{=1}$ ⇨ $\overrightarrow{AB}.\ \overrightarrow{BC} = a\,b\,\cos\beta$

En utilisant la définition du produit scalaire on obtient :

$$\overrightarrow{AB}.\,\overrightarrow{BC} = \left\|\overrightarrow{AB}\right\| \times \left\|\overrightarrow{BC}\right\| \times \cos\left(\overrightarrow{AB}\;\overrightarrow{CD}\right) = a\,b\,\cos\beta$$

Q4) $$\overrightarrow{AB} \wedge \overrightarrow{BC} = \begin{pmatrix} a\cos\alpha \\ a\sin\alpha \\ 0 \end{pmatrix}_{B_1} \wedge \begin{pmatrix} b\times(\cos\beta\cos\alpha - \sin\beta\sin\alpha) \\ b\times(\cos\beta\sin\alpha + \sin\beta\cos\alpha) \\ 0 \end{pmatrix}_{B_1}$$

$$a\cos\alpha \qquad b\times(\cos\beta\cos\alpha - \sin\beta\sin\alpha)$$

$$\Rightarrow \overrightarrow{AB} \wedge \overrightarrow{BC} = \begin{pmatrix} 0 \\ 0 \\ a\cos\alpha \times b\times(\cos\beta\sin\alpha + \sin\beta\cos\alpha) - a\sin\alpha\times b\times(\cos\beta\cos\alpha - \sin\beta\sin\alpha) \end{pmatrix}_{B_1}$$

$$\Rightarrow \overrightarrow{AB} \wedge \overrightarrow{BC} = a\,b\times\left(\cancel{\cos\alpha\cos\beta\sin\alpha} + \sin\beta\cos^2\alpha - \cancel{\sin\alpha\cos\beta\cos\alpha} + \sin\beta\sin^2\alpha\right)\vec{z}_1$$

$$\Rightarrow \overrightarrow{AB} \wedge \overrightarrow{BC} = a\,b\times\sin\beta\times\underbrace{\left(\cos^2\alpha + \sin^2\alpha\right)}_{=1}\vec{z}_1 \Rightarrow \overrightarrow{AB} \wedge \overrightarrow{BC} = a\,b\,\sin\beta\,\vec{z}_1$$

En utilisant la définition du produit vectoriel on obtient :

$$\overrightarrow{AB}.\,\overrightarrow{BC} = \left\|\overrightarrow{AB}\right\| \times \left\|\overrightarrow{BC}\right\| \times \sin\left(\overrightarrow{AB}\;\overrightarrow{CD}\right)\vec{z}_1 = a\,b\,\sin\beta\,\vec{z}_1$$

Q5) $$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} a\cos\alpha \\ a\sin\alpha \\ 0 \end{pmatrix}_{B_1} + \begin{pmatrix} b\times(\cos\beta\cos\alpha - \sin\beta\sin\alpha) \\ b\times(\cos\beta\sin\alpha + \sin\beta\cos\alpha) \\ 0 \end{pmatrix}_{B_1}$$

$$\Rightarrow \overrightarrow{AC} = \begin{pmatrix} a\cos\alpha + b\times(\cos\beta\cos\alpha - \sin\beta\sin\alpha) \\ a\sin\alpha + b\times(\cos\beta\sin\alpha + \sin\beta\cos\alpha) \\ 0 \end{pmatrix}_{B_1}$$

AN : ***$\alpha = 0°$ $\cos\alpha = 1$ et $\sin\alpha = 0$ $\beta = 90°$ $\cos\beta = 0$ et $\sin\beta = 1$***

$$\Rightarrow \overrightarrow{AC} = \begin{pmatrix} a + b\times(0-0) \\ 0 + b\times(0 + 1\times 1) \\ 0 \end{pmatrix}_{B_1} \Rightarrow \overrightarrow{AC} = \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}_{B_1} \Rightarrow \left\|\overrightarrow{AC}\right\| = \sqrt{a^2 + b^2}$$

Normal car le triangle ***(A B C)*** est alors rectangle en ***B***