VITESSE, ACCELERATION (CAS DE LA ROTATION)

Q1)

$$\overrightarrow{\Omega_{2/1}} = \dot{\alpha} \times \vec{z}_i \qquad \overrightarrow{\Omega_{3/2}} = \dot{\beta} \times \vec{z}_i \qquad \overrightarrow{\Omega_{3/1}} = \overrightarrow{\Omega_{3/2}} + \overrightarrow{\Omega_{2/1}} = \left(\dot{\alpha} + \dot{\beta}\right) \times \vec{z}_i$$

Q2)

- *Le mouvement **2/1** est une simple rotation de centre **A** donc :* $\overrightarrow{V_{B\in 2/1}} = a \times \dot{\alpha} \times \vec{y}_2$

- *Dérivation du vecteur position :* $\overrightarrow{V_{B\in 2/1}} = \left(\dfrac{d\overrightarrow{AB}}{dt}\right)_{/1} = \left(\dfrac{d\left(a\,\vec{x}_2\right)}{dt}\right)_{/1} = a \times \left(\dfrac{d\,\vec{x}_2}{dt}\right)_{/1}$

$$\Rightarrow \quad \overrightarrow{V_{B\in 2/1}} = +a\,\dot{\alpha}\,\vec{y}_2$$

- *Changement de point :* $\overrightarrow{V_{B\in 2/1}} = \overrightarrow{V_{A\in 2/1}} + \overrightarrow{BA} \wedge \overrightarrow{\Omega_{2/1}} = -a\,\vec{x}_2 \wedge \dot{\alpha}\,\vec{z}_2$

$\vec{0}$
car ***2/1*** est une rotation de centre ***A***

$$\Rightarrow \quad \overrightarrow{V_{B\in 2/1}} = +a\,\dot{\alpha}\,\vec{y}_2$$

Q3)

- *Dérivation du vecteur position :* $\overrightarrow{V_{C\in 3/1}} = \left(\dfrac{d\overrightarrow{AC}}{dt}\right)_{/1} = \left(\dfrac{d\left(\overrightarrow{AB} + \overrightarrow{BC}\right)}{dt}\right)_{/1}$

(sans utiliser la formule de Boor)

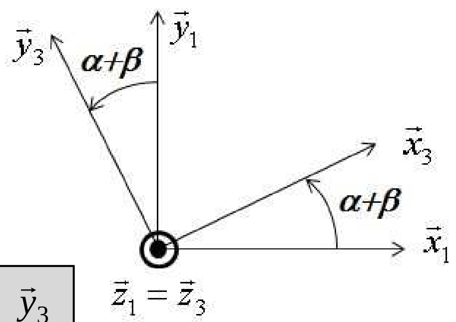
$$\Rightarrow \quad \overrightarrow{V_{C\in 3/1}} = \left(\frac{d\overrightarrow{AB}}{dt}\right)_{/1} + \left(\frac{d\overrightarrow{BC}}{dt}\right)_{/1} = \left(\frac{d\left(a\,\vec{x}_2\right)}{dt}\right)_{/1} + \left(\frac{d\left(b\,\vec{x}_3\right)}{dt}\right)_{/1} = +a\,\dot{\alpha}\,\vec{y}_2 + b \times \left(\frac{d\vec{x}_3}{dt}\right)_{/1}$$

$$\Rightarrow \quad \overrightarrow{V_{C\in 3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 + b \times \frac{d}{dt}\left[\cos(\alpha+\beta)\,\vec{x}_1 + \sin(\alpha+\beta)\,\vec{y}_1\right]_{/1}$$

$$\Rightarrow \quad \overrightarrow{V_{C\in 3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 + b \times \begin{pmatrix} -\left(\dot{\alpha}+\dot{\beta}\right) \times \sin(\alpha+\beta) \\ +\left(\dot{\alpha}+\dot{\beta}\right) \times \cos(\alpha+\beta) \\ 0 \end{pmatrix}_1$$

$$\Rightarrow \quad \overrightarrow{V_{C\in 3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 + b \times \left(\dot{\alpha}+\dot{\beta}\right) \times \begin{pmatrix} -\sin(\alpha+\beta) \\ +\cos(\alpha+\beta) \\ 0 \end{pmatrix}_1$$

$\vec{y}_3$



$$\Rightarrow \quad \overrightarrow{V_{C\in 3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 + b\left(\dot{\alpha}+\dot{\beta}\right)\vec{y}_3$$

- *Dérivation du vecteur position :* (en utilisant la formule de Boor)

$$\overrightarrow{V_{C\in 3/1}} = \left(\frac{d\,\overrightarrow{AB}}{dt}\right)_{/1} + \left(\frac{d\,\overrightarrow{BC}}{dt}\right)_{/1}$$

$$\left(\frac{d\,\overrightarrow{AB}}{dt}\right)_{/1} = +a\,\dot{\alpha}\,\vec{y}_2$$

$$\left(\frac{d\,\overrightarrow{BC}}{dt}\right)_{/1} = \left(\frac{d\,\overrightarrow{BC}}{dt}\right)_{/2} + \overrightarrow{BC} \wedge \overrightarrow{\Omega_{1/2}}$$

$$\left(\frac{d\,\overrightarrow{BC}}{dt}\right)_{/2} = b \times \left(\frac{d(\vec{x}_3)}{dt}\right)_{/2} = +b\,\dot{\beta}\,\vec{y}_3$$

$$\Rightarrow \overrightarrow{V_{C\in 3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 + \left(\frac{d\,\overrightarrow{BC}}{dt}\right)_{/2} + b\,\vec{x}_3 \wedge (-\dot{\alpha}\,\vec{z}_3)$$

$$\Rightarrow \overrightarrow{V_{C\in 3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 + b\,\dot{\beta}\,\vec{y}_3 + b\,\dot{\alpha}\,\vec{y}_3$$

$$\Rightarrow \overrightarrow{V_{C\in 3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 + b\left(\dot{\alpha} + \dot{\beta}\right)\vec{y}_3$$

- *Changement de point :*

$$\overrightarrow{V_{C\in 3/1}} = \overrightarrow{V_{B\in 3/1}} + \overrightarrow{CB} \wedge \overrightarrow{\Omega_{3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 + (-b\,\vec{x}_3) \wedge \left(\dot{\alpha} + \dot{\beta}\right) \times \vec{z}_i$$

$$\overrightarrow{V_{B\in 3/1}} = \overrightarrow{V_{B\in 3/2}} + \overrightarrow{V_{B\in 2/1}}$$

$\overrightarrow{V_{B\in 3/2}} = \vec{0}$ car ***3/2*** est une rotation de centre ***B***

$\overrightarrow{V_{B\in 2/1}} = +a\,\dot{\alpha}\,\vec{y}_2$

$$\overrightarrow{V_{C\in 3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 - b\,\vec{x}_3 \wedge \left(\dot{\alpha} + \dot{\beta}\right)\vec{z}_3$$

$$\overrightarrow{V_{C\in 3/1}} = +a\,\dot{\alpha}\,\vec{y}_2 + b\left(\dot{\alpha} + \dot{\beta}\right)\vec{y}_3$$

Q4)

- *Accélération absolue du point **B** :*

$$\overrightarrow{\Gamma_{B\in 2/1}} = \left(\frac{d\,\overrightarrow{V_{B\in 2/1}}}{dt}\right)_{/1} = \left(\frac{d\,(a\,\dot{\alpha}\,\vec{y}_2)}{dt}\right)_{/1} = a \times \left(\frac{d\,(\dot{\alpha}\,\vec{y}_2)}{dt}\right)_{/1}$$

$$\Rightarrow \overrightarrow{\Gamma_{B\in 2/1}} = a \times \left[\ddot{\alpha}\,\vec{y}_2 + \dot{\alpha} \times \left(\frac{d\,(\vec{y}_2)}{dt}\right)_{/1}\right] = a \times \left[\ddot{\alpha}\,\vec{y}_2 + \dot{\alpha} \times (-\dot{\alpha}\,\vec{x}_2)\right]$$

$$\Rightarrow \overrightarrow{\Gamma_{B\in 2/1}} = -a\,\dot{\alpha}^2\,\vec{x}_2 + a\,\ddot{\alpha}\,\vec{y}_2$$

$-a\,\dot{\alpha}^2\,\vec{x}_2$: Accélération centripète

$a\,\ddot{\alpha}\,\vec{y}_2$: Accélération tangentielle

- *Accélération absolue du point **C** :*

$$\overrightarrow{\Gamma_{C\in 3/1}} = \left(\frac{d\,\overrightarrow{V_{C\in 3/1}}}{dt}\right)_{/1} = \left(\frac{d\,\left(a\,\dot{\alpha}\,\vec{y}_2 + b\left(\dot{\alpha}+\dot{\beta}\right)\vec{y}_3\right)}{dt}\right)_{/1}$$

$$\Rightarrow \overrightarrow{\Gamma_{C\in 3/1}} = a\times\left(\ddot{\alpha}\,\vec{y}_2 + \dot{\alpha}\times\left(\frac{d\,(\vec{y}_2)}{dt}\right)_{/1}\right) + b\times\left(\left(\ddot{\alpha}+\ddot{\beta}\right)\vec{y}_3 + \left(\dot{\alpha}+\dot{\beta}\right)\times\left(\frac{d\,(\vec{y}_3)}{dt}\right)_{/1}\right)$$

$$\Rightarrow \overrightarrow{\Gamma_{C\in 3/1}} = a\times\left[\ddot{\alpha}\,\vec{y}_2 + \dot{\alpha}\times\left(-\dot{\alpha}\,\vec{x}_2\right)\right] + b\times\left[\left(\ddot{\alpha}+\ddot{\beta}\right)\vec{y}_3 + \left(\dot{\alpha}+\dot{\beta}\right)\times\left(-\left(\dot{\alpha}+\dot{\beta}\right)\vec{x}_3\right)\right]$$

$$\Rightarrow \boxed{\overrightarrow{\Gamma_{C\in 3/1}} = a\times\left(\ddot{\alpha}\,\vec{y}_2 - \dot{\alpha}^2\,\vec{x}_2\right) + b\times\left[\left(\ddot{\alpha}+\ddot{\beta}\right)\vec{y}_3 - \left(\dot{\alpha}+\dot{\beta}\right)^2\vec{x}_3\right]}$$